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Numerical study on transverse asymmetry in the temperature profile of a regenerator in a pulse tube cooler

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Abstract

Transverse asymmetry in the temperature profile of the regenerator in a Stirling-type pulse tube cooler as observed in experiments was analysed in a numerical study. The asymmetry was reproduced using a one-dimensional model of the cooler where the regenerator was modelled using two identical parallel regenerator channels. The asymmetry was caused by a circulating flow that was superimposed on the oscillating flow. The primary mechanism driving the circulating flow was due to the wave form of the pressure difference between the ends of the regenerator and the dependence of the instantaneous mass flow rate on the pressure difference and temperature.

Keywords: Pulse tube cooler; Regenerator; Temperature profile; Transverse asymmetry; Circulating flow; Streaming

1. Introduction

In contrast to the traditional regenerative cryocoolers, such as the Stirling– and Gifford–McMahon (GM)-cryocoolers, the pulse tube cryocooler (PTC) operates without a cold moving displacer. This feature leads to increased reliability, lower manufacturing costs and reduced mechanical vibrations at the cold head. Stirling-type high frequency PTCs are particularly attractive, since they can be operated with rubbing-free linear compressors that significantly increase the maintenance free operation time of the cooling system (see [1] for a recent review). At present, there is growing interest in large Stirling-type PTCs with electrical input power higher than 4 kW for potential use in gas liquefaction and power applications of superconductors [2–5]. For hydrodynamic reasons, the up-scaling of the cooler size for high power leads to regenerators with large cross section areas and low aspect ratios. Such a geometry can give rise to unwanted temperature inhomogeneities in the regenerator, as has been recently observed [3–6].

Temperature differences up to 160 K transverse to the main flow direction have been measured in the temperature distribution in the regenerator of a high-power Stirling-type pulse tube cooler targeted for 80 W cooling at 25–30 K [4,5]. In the experiments it appeared that the transverse temperature asymmetry was initiated when the input power to the cooler, and hence the oscillating mass flow through the regenerator, exceeded a temperature dependent critical value. Dependent on the input power and the wire diameter and material of the wire screen mesh in the regenerator the asymmetry would then need on the order of magnitude 1 h of cooler operation to evolve to the fully asymmetric temperature distribution. The transverse asymmetry in the temperature profile was shown [4,5] to considerably reduce both the available cooling power and the efficiency of the PTC and to limit the obtainable no-load temperature. It was found that the transverse asymmetry and its detrimental effects on the performance of the PTC could be reduced by increasing the transverse heat conductivity of the regenerator matrix by replacing

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Nomenclature

C_{fd}	form drag coefficient	p	pressure (Pa)
C_{sf}	skin friction coefficient	$\bar{p}$	space averaged pressure (Pa)
$\dot{E}_{est}$	estimated rate of energy transport (W)	Re	Reynolds number
$\dot{E}_{mat}$	rate of heat transfer to matrix (W)	t	time (s)
$\dot{E}_{regloss}$	regenerator energy flux loss (W)	T	temperature (K)
Δp_{fric}	pressure difference due to friction in the regenerator (Pa)	$\overline{T}$	space averaged temperature (K)
		$\overline{V}$	cup velocity (m/s)
Δp_{reg}	instantaneous pressure difference between ends of regenerator (Pa)	w	open mesh width (m)
		$\oint$	integral in time over one cycle of the machine
$\widetilde{\Delta p_{reg}}$	wave form of the time variation of pressure difference between ends of regenerator (Pa)		
		Greek symbols	
f	operating frequency (1/s)	ν	kinematic viscosity (m^2/s)
n	number of wire screens	μ	dynamic viscosity (kg/(m s))
$\dot{m}$	mass flow rate (kg/s)	ρ	density (kg/m^3)

some of the stainless steel wire screens in the matrix with copper wire screens.

The basic geometry of the problem is illustrated in Fig. 1. A cylindrical regenerator is subjected to the oscillating flow in the PTC. Three Pt-100 temperature sensors were placed at the midplane of the regenerator interspaced by 120 degrees along the periphery of the regenerator canister. The sensors were expected to show identical temperatures during operation. But at some point in time during the cool down phase of the PTC the temperatures measured by the sensors began to diverge. Once the temperatures measured by the sensors scattered the sensors did not converge back to having the same temperature until the PTC was turned off.

The orientation of the asymmetry was observed to vary between different experimental runs with the same experimental setup, i.e. it varied from experiment to experiment which of the three temperature sensors that measured the highest temperature and which of the temperature sensors that measured the lowest temperature. In some experiments it was even observed that the asymmetry appeared to rotate very slowly with a period of the order of magnitude 5 h. Due to these observations the transverse asymmetry in the temperature profile was not believed to be caused only by geometric asymmetry in the experimental setup.

In regenerators there are a number of mechanisms for energy transport transverse to the main flow direction that would be expected to work against transverse asymmetry in a matrix temperature profile. Prime amongst these mechanisms are:

- Conduction in the wires of the regenerator matrix. This conduction will be more powerful in wire screen matrices than in metal felt matrices, because the wires run unbroken across the entire cross section of the regenerator in the wire screens. The magnitude of the energy

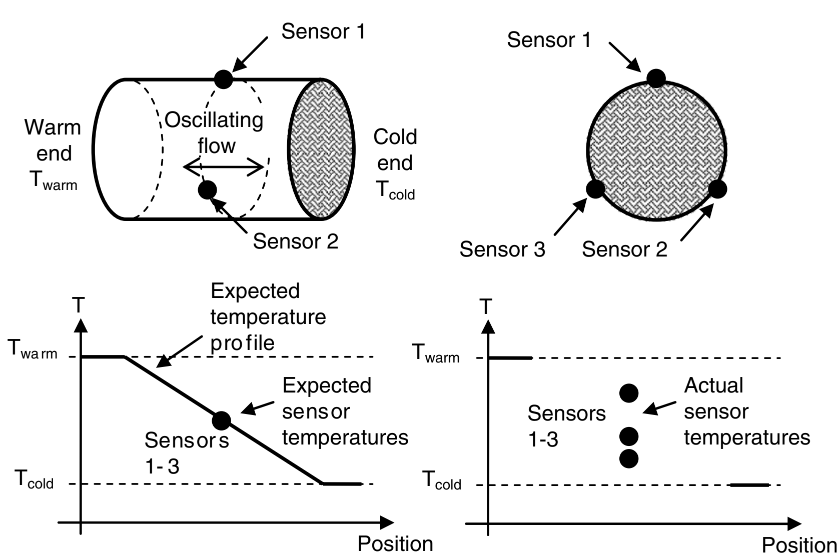

Fig. 1. Basic geometry of problem.

transport by conduction in the matrix wires will be proportional to the transverse temperature gradient, the thermal conductivity of the matrix material, and to the amount of matrix material in the regenerator. The transverse temperature gradient corresponding to a given temperature difference between opposing sides of the regenerator is inversely proportional to the diameter of the regenerator. The conduction in the wires will hence be most effective in removing transverse temperature differences in regenerators with small diameters. As mentioned above it has been verified experimentally [4,5] that replacing a fraction of the stainless steel wire screens in a regenerator by copper wire screens can reduce the magnitude of a transverse asymmetry in the matrix temperature profile.

- Molecular conduction in the gas in the regenerator. The thermal conductivity of helium is two orders of magnitude smaller than the thermal conductivity of stainless steel matrix wires. For regenerator matrices, which typically have porosities between 50% and 80%, the energy transport by conduction in the gas is therefore expected to be much smaller than the conduction in the wires.
- Enhanced transverse energy transport due to the turbulence and mixing in the flow through the porous matrix of the regenerator. Gedeon and Wood [7] have derived a correlation for the axial conduction enhancement due to turbulence in wire screen regenerators. This correlation predicts that the axial energy transport due to turbulence is between one and two orders of magnitude larger than the axial energy transport due to molecular conduction in the gas in the regenerator of the pulse tube cooler from the experimental study described in Refs. [4,5]. If the energy transport transverse to the flow direction due to turbulence is of similar magnitude, then this energy transport could be of the same order of magnitude as the conduction in the wires in regenerator with stainless steel wire screens.
- Bulk cross flow induced by the transverse temperature asymmetry. The gas density in a regenerator is almost inversely proportional to the temperature. If a parallel flow with uniform temperature and velocity distributions enters a regenerator matrix where there is transverse asymmetry in the temperature distribution, then part of the mass flow must flow from the hot side towards the cold side inside the regenerator. The magnitude of the energy transport carried by cross flow is difficult to estimate.

Axial energy transport can also smooth out the asymmetry. Imagine, for instance, that the regenerator canister was emptied so that only two identical wires remained in the canister and that these wires were at the same axial position in the regenerator. If bursts of hot and cold gas were alternately sent through the regenerator canister then surely the temperatures of the wires would soon be identical regardless of any differences in their initial temperatures.

The aims of this study were (1) to reproduce the experimentally observed transverse asymmetry in the matrix temperature profile in numerical simulations, and (2) to identify the mechanisms that can cause and sustain a transverse asymmetry in the temperature profile of a regenerator with no transverse geometric asymmetry. The study was performed in two stages using two separate numerical models.

As the first stage of the study we used a complete simulation model of a PTC, where the regenerator was divided into two parallel regenerator channels, to reproduce the transverse temperature asymmetry in the regenerator. We found that the cause of the asymmetry was a circulating flow in the closed loop formed by the two parallel regenerator channels and the manifold volumes at the ends of the regenerator. This circulating flow was superimposed on the oscillating flow through the regenerator. The circulating flow (or streaming) amplified any small transverse asymmetry in the regenerator temperatures.

As the second stage we used a separate, simplified model of one regenerator channel to identify the mechanism driving the circulating flow, i.e. to study the influence of the regenerator matrix temperature on the mass flow predicted by the equations for the regenerator. We found that the circulating flow was due to the shape of the pressure difference wave, $\widetilde{\Delta p_{\mathrm{reg}}}$, that drives flow through the regenerator, and the dependence of the instantaneous mass flow rate through the regenerator on the instantaneous pressure difference, Δp_{reg}, and the temperature. We also found that the temperature oscillation in the regenerator had a small amplifying effect on the circulating flow and that the oscillation of the pressure in the regenerator damped the circulating flow. The net result was that a regenerator channel would draw in mass from the cold end of the regenerator if the temperature in the channel decreased, and conversely draw mass from the hot end of the regenerator if the temperature increased.

2. Method

In the first stage of the study we used a complete model of a PTC to reproduce the transverse asymmetry in the regenerator temperatures and the circulating flow which caused the transverse asymmetry. In the second stage of the study we used a separate, simple model of one regenerator channel to study the mechanisms that can drive a circulating flow.

2.1. The complete pulse tube cooler model

The complete PTC model was used to reproduce the experimentally observed transverse regenerator temperature asymmetry in numerical simulations.

The complete PTC model was built using the control volume based approach described by Andersen et al. [9,10] for modelling oscillating, compressible flow which is primarily one dimensional. This modelling approach has been successfully validated for both Stirling machines and pulse tube coolers [9,10], and has been used specifically to study regenerators in Stirling engines [11,12]. The model used in this study has been verified by Andersen [10] to produce results in good agreement with the experimental data and with another pulse tube cooler model constructed in the state of the art simulation software *Sage* of Gedeon [8].

The complete PTC model was built so that the regenerator could either be modelled as a single regenerator channel or be divided into two parallel regenerator channels, each with half the cross sectional area of the single regenerator channel. The components of the PTC included in the computational domain are shown in Fig. 2 for the case where the regenerator is divided into two parallel channels.

When two regenerator channels were used they were not connected in the transverse direction. Hence the two channels functioned as two identical, parallel, and completely separate regenerators that shared the same boundary conditions. Because the two channels were completely separate the mechanisms for transverse energy transport that normally work against transverse temperature asymmetry were not included in the model. The model thus represents an extreme case: it represents the situation which we would expect to be least stable.

The model with two parallel regenerator channels was verified to give results for symmetric solutions that were identical to the results obtained with one regenerator channel.

In the regenerator of the complete PTC model the flow friction was calculated using the empirical correlation described by Thomas and Pittman [13] with coefficients for data by Gedeon and Wood [7] for flow through wire screen matrices:

$$\Delta p_{\text{fric}} = \left(C_{\text{fd}} + \frac{C_{\text{sf}}}{Re}\right) \cdot n \cdot \frac{1}{2} \cdot \rho \cdot \overline{V}^2 \cdot \frac{\overline{V}}{|\overline{V}|},$$
$$Re = \frac{w \cdot \overline{V}}{\nu}, \quad C_{\text{fd}} = 0.5274, \quad C_{\text{sf}} = 68.556. \tag{1}$$

The regenerator had a porosity of 64.4% and consisted of stainless steel wire screens with a wire diameter of 30 μm, corresponding to an open mesh width of $w = 41.7$ μm.

2.2. Reproducing the transverse asymmetry

A simulation using the complete PTC model with two parallel regenerator channels was started as an initial value problem with a slightly asymmetric initial temperature distribution to see if the asymmetry would increase or decrease with time. The initial asymmetry was introduced by modifying a symmetric solution by making a notch in the axial matrix temperature profile in one of the parallel channels. By performing the experiment twice with the notch in the temperature profile placed in different channels it was also verified that the complete PTC model itself was symmetric.

In the complete PTC model the two regenerator channels and the manifold volumes at the ends of the regenerator formed a closed loop flow path. Gedeon [14] has previously shown that circulating flows are to be expected in oscillating flow in machines with closed loop flow paths that are not geometrically symmetrical. In the complete PTC model, however, the closed loop flow path formed by the two regenerator channels had perfect geometrical symmetry. The only asymmetry that could drive a circulating flow in the numerical experiments was hence the transverse asymmetry in the temperature profiles of the two regenerator channels.

A circulating flow was found and it was tested if the circulating flow could be the main energy transport mechanism increasing the asymmetry. This was tested by comparing the energy transport by the circulating flow to the rates of change in the amounts of energy stored in the matrices of the regenerator channels. The energy transport by the circulating flow was estimated as the circulating mass flow times the enthalpy change in the gas when it travelled from the inflow end of a regenerator channel to the outflow end.

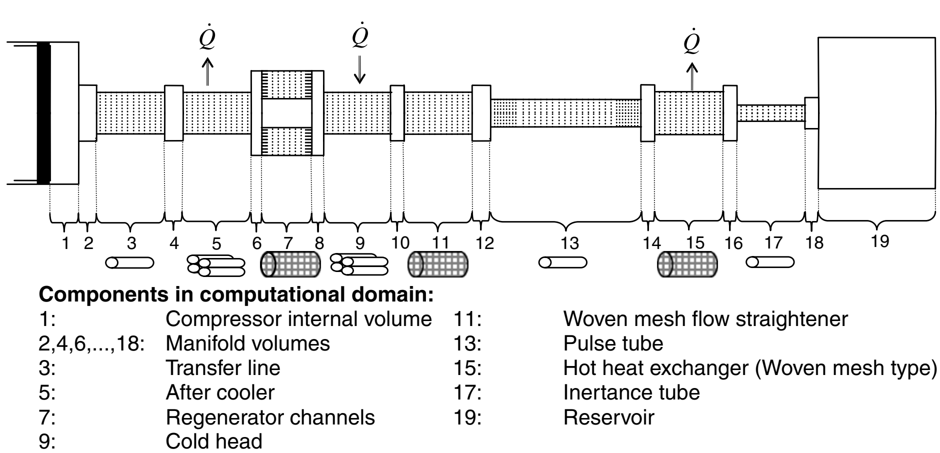

Fig. 2. Computational domain of pulse tube cooler model.

2.3. The separate, simple model of a regenerator channel

A separate, simple model of a single regenerator channel was used to isolate and study the sensitivities to temperature changes of the individual mechanisms that influence the cycle averaged mass flow rates through a regenerator channel.

In the simple regenerator model the regenerator channel was lumped into a single control volume. The mass flow rate through the regenerator was calculated from the average conditions in this control volume and from the pressure difference between the ends of the regenerator. The mass flow was calculated by setting the pressure drop across the regenerator, Δp_{reg}, equal to Δp_{fric} from Eq. (1) and solving for $\overline{V}$. The simple regenerator model thus assumed quasi-steady flow and ignored effects due to the inertia of the gas. The pressure difference between the ends of the regenerator and the average pressure in the regenerator were prescribed explicitly, so that they could be either simple analytical functions of time or be generated by use of data exported from the complete PTC model or from the *Sage* model of the PTC.

Because there was only one control mass for the regenerator matrix inside the single control volume for the regenerator it was only the average matrix temperature which was known in the simple regenerator model. The matrix temperature could either be assumed constant or be simulated using an energy balance for a lumped control mass. The average temperature difference between the gas and the regenerator matrix was assumed constant but dependent on the flow direction. The average gas temperature in the matrix, which was needed to evaluate Eq. (1), could then be calculated from the matrix temperature and the temperature difference between the gas and the matrix. The gas temperatures outside the ends of the regenerator were set equal to 300 K and 60 K. When matrix temperatures were modelled as dynamic, in order to take into account the effect of matrix temperature oscillations, these temperatures were used as the inlet temperatures to the single control volume of the regenerator. The outflow temperature was calculated as the temperature outside the outlet end +/− the constant temperature difference between gas and matrix depending on the flow direction. The method used for calculating the inlet and outlet temperatures only affected the magnitude of the matrix temperature oscillations because it was the before mentioned average gas temperature in the control volume which was used when evaluating Eq. (1).

The calculations performed with the simple regenerator model hence depended mainly on Eq. (1), the boundary conditions, and the gas viscosity used in the calculations. The temperature dependent gas viscosity only varied during individual cycles when the gas temperature varied, i.e. when the matrix temperatures were non-constant and/or there was a finite temperature difference between the gas and the matrix.

The tendencies observed for the single control volume in the simple regenerator model should be the same as the tendencies of individual control volumes in a more complex model, such as the complete PTC model.

2.4. Identifying mechanisms which can contribute to the circulating flow

The simple regenerator model was used for studying how changes in the regenerator matrix temperature influenced the cycle averaged mass flow rate, $f\cdot\oint\dot{m}$, when there was an oscillating mass flow through the regenerator. In practice all the phenomena in the oscillating flow through the regenerator are coupled. But in order to understand the mechanisms driving and opposing the circulating flow, the oscillating flow was split into simpler phenomena which each influence $f\cdot\oint\dot{m}$ through a regenerator channel.

Firstly, it is clear that the wave form of the time dependent pressure difference between the ends of a regenerator channel, $\widetilde{\Delta p}_{\mathrm{reg}}$, influenced $f\cdot\oint\dot{m}$. $\widetilde{\Delta p}_{\mathrm{reg}}$ was identical for the parallel regenerator channels. But if the contribution from $\widetilde{\Delta p}_{\mathrm{reg}}$ to $f\cdot\oint\dot{m}$ through a regenerator channel depended on temperature, then the contributions from $\widetilde{\Delta p}_{\mathrm{reg}}$ to $f\cdot\oint\dot{m}$ would be different for different channels with different temperatures.

Secondly, the oscillation of the absolute pressure, $\overline{p}$, in the regenerator also influenced $f\cdot\oint\dot{m}$. In the studied PTC $\overline{p}$ in the regenerator oscillated with an amplitude that was roughly 12% of the cyclic mean pressure $f\cdot\oint\overline{p}$. The pressure oscillation had a small phase lead of approximately 20 degrees over the pressure difference oscillation, and hence over the mass flow oscillation. The pressure was thus above average when the flow was towards the cold end of the regenerator and below average when the flow was towards the hot end. In Eq. (1) it can be seen that in the limit where Δp_{fric} approaches zero, the term C_{fd} becomes insignificant. In this case the dependence of Re on ρ cancels the dependence of Δp_{fric} on ρ, so that the velocity becomes:

$$\overline{V} = \frac{2\cdot w}{C_{\mathrm{sf}}\cdot\mu\cdot n}\cdot\Delta p_{\mathrm{fric}} \tag{2}$$

The volumetric flow rate therefore becomes independent of $\overline{p}$ as μ is essentially independent of $\overline{p}$. Since the density of the gas is proportional to the pressure the mass flow rate then also becomes proportional to $\overline{p}$. For the range of Δp_{reg} in the PTC model it remains true that the mass flow rate, as computed by use of Eq. (1), was nearly proportional to $\overline{p}$. Because the oscillation in $\overline{p}$ was almost in phase with the oscillation in Δp_{reg}, and hence with the mass flow rate, the oscillation in $\overline{p}$ increased the cycle averaged mass flow rate towards the cold end of a regenerator channel. Because the pressure oscillation was the same for parallel regenerator channels it is, as before, the derivative with respect to $\overline{T}$ of the contribution to $f\cdot\oint\dot{m}$ which is of interest, because this derivative tells us what happens when the temperatures in two parallel regenerator channels are different.

Thirdly, the oscillation in time of the space averaged temperature, $\overline{T}$, in the regenerator also influenced $f \cdot \oint \dot{m}$ through a regenerator channel. The temperature oscillation, which had an amplitude of approximately 0.7 K in the studied PTC, was partly due to the finite heat capacity of the regenerator matrix, and partly due to imperfect heat transfer between gas and matrix and the oscillation of the flow direction. The combined effect of the matrix temperature oscillations and of the imperfect heat transfer was that $\overline{T}$ was a little higher when gas flowed towards the cold end of the regenerator than when gas flowed towards the hot end.

The temperature oscillation caused a slight increase in μ and a slight decrease in ρ when the flow was towards the cold end of the regenerator compared to when the flow was towards the hot end. The oscillation in temperature thus had the effect of decreasing the cycle averaged mass flow through a regenerator channel towards the cold end of the regenerator.

Again it is the derivative of the contribution to $f \cdot \oint \dot{m}$ due to the oscillation in $\overline{T}$ which is of particular interest with respect to the circulating flow.

2.5. Testing the possible contributions to the circulating flow

The contribution to the cycle averaged mass flow rate due to the shape of the pressure difference wave was first studied by mapping how the instantaneous mass flow rate, $\dot{m}$, through the regenerator depended on instantaneous pressure difference between the ends of the regenerator, Δp_{reg}, in the temperature interval from 50 to 350 K.

The simple regenerator model was then used to integrate $\dot{m}$ during cycles to find the cycle averaged mass flow rates for different pressure difference waves for matrix temperatures between 50 and 350 K. In each of these integrated cycles $\overline{p}$ and the matrix temperature were kept constant, and it was assumed that $\overline{T}$ was equal to the constant regenerator matrix temperature. The calculations were performed for ideal gas helium with temperature dependent viscosity at $\overline{p} = 2$ MPa. The derivative of $f \cdot \oint \dot{m}$ with respect to $\overline{T}$ was then calculated from the results.

The contribution from the oscillation in $\overline{p}$ to $f \cdot \oint \dot{m}$ was studied by including the oscillation in oscillations in $\overline{p}$ into the simple regenerator model. We then repeated the integrations of the instantaneous mass flow rates through a regenerator channel during cycles to determine the value of $f \cdot \oint \dot{m}$ for different matrix temperatures. The only difference between these integrations and the integrations for constant $\overline{T}$ and $\overline{p}$ in the regenerator was that $\overline{p}$ oscillated in time. Again the derivative of $f \cdot \oint \dot{m}$ with respect to $\overline{T}$ was calculated from the results.

Finally, the contribution from the oscillation in $\overline{T}$ was studied by also including the matrix temperature oscillations into the simple regenerator model, and calculating $f \cdot \oint \dot{m}$ for different initial values of $\overline{T}$. In these calculations it was assumed, based on observations of the solutions to the complete PTC model, that the constant temperature difference between the gas and the matrix was 0.3 K. The derivative of $f \cdot \oint \dot{m}$ with respect to $\overline{T}$ was also calculated for these results.

The results from the simple regenerator model depend strongly on Eq. (1) and the viscosity of the gas is used in Eq. (1). As the final test the calculations for oscillating $\overline{T}$ and $\overline{p}$ were repeated with a constant viscosity equal to the viscosity at 200 K.

3. Results and discussion

3.1. Reproducing the circulating flow using the complete pulse tube cooler model

The results from the experiment where a simulation using the complete PTC model was started with a slightly asymmetric initial temperature distribution, i.e. with a hand made notch in the temperature profile in one channel, are shown in Fig. 3 shows that the slightly asymmetric solution was indeed unstable. After a fast initial transient where the notch in the temperature profile was smoothed out by axial energy transport the temperature profiles in the two regenerator channels began to diverge at an accelerating rate. The divergence continued until the temperature profiles were very asymmetric. At the asymmetric periodic steady state solution (number of cycles, $f \cdot t$, >30 000 in Fig. 3) the mechanisms that caused the asymmetry were balanced by axial energy transport mechanisms.

Inspection of the mass flow rates in the two channels in the simulation revealed a circulating flow (streaming) superimposed on the oscillating flow. The mass flow rate of the circulation increased as the asymmetry developed and it increased the total flux loss through the regenerator by up to a factor of 5, as shown in Fig. 4. The increase in the regenerator loss agrees with the experimental observa-

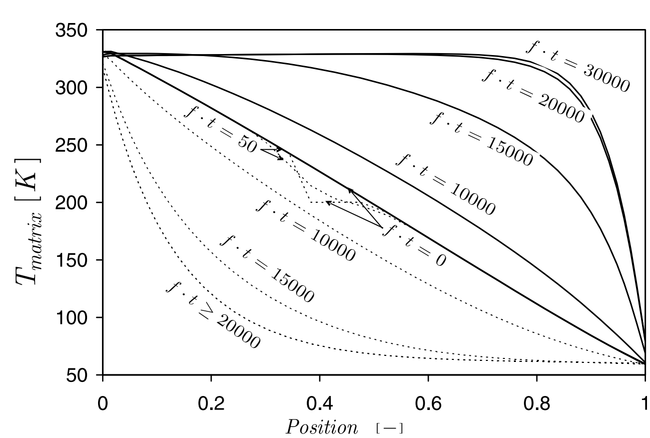

Fig. 3. The evolution in time of the asymmetry in the temperature profiles in the two regenerator channels. At $f \cdot t = 0$ a notch is introduced in the temperature profile of one channel.

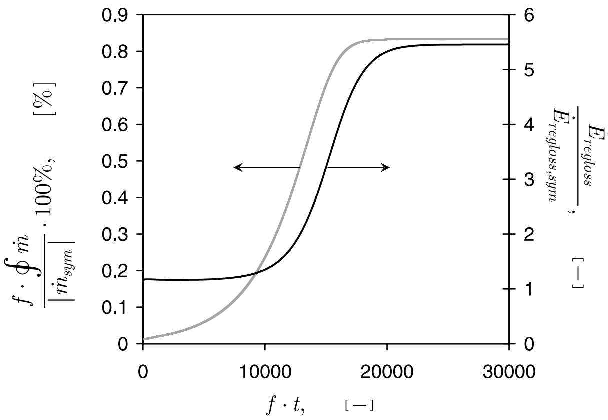

Fig. 4. The evolution in time of the DC flow plotted in percent of the regenerator mass flow amplitude for the symmetric solution, and the evolution of the total energy flux loss through the regenerator, plotted relative to the total energy flux loss for the symmetric solution.

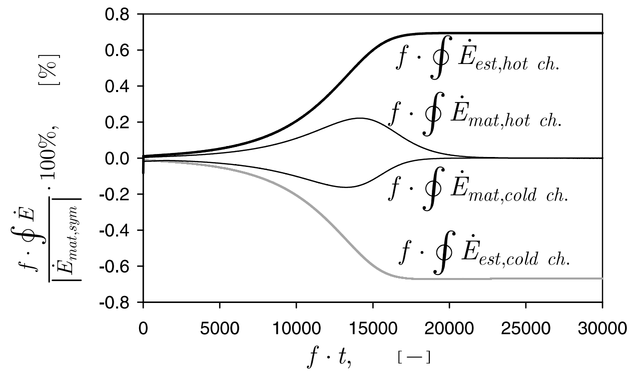

Fig. 5. The time evolutions of the rates of change of energy stored in the matrices of the regenerator channels and of the estimates for the rates of energy transfer by the circulating flow. All quantities have been made dimensionless with the amplitude of the oscillation in the amount of energy stored in the regenerator matrix in the symmetric solution.

tion that the performance of the PTC suffered when the regenerator matrix temperatures became asymmetric.

The circulating flow drew mass from the cold end of the regenerator into the channel with the lowest average temperature and conversely drew mass from the hot end of the regenerator into the channel with the highest average temperature. The circulating flow removed energy from the coldest channel because the circulating gas was heated when it travelled from the cold end to the hot end of the regenerator. Conversely, the circulating flow transported energy into the warmest regenerator channel, because the circulating gas was cooled on its way from the hot to the cold end. The circulating flow thus amplified the asymmetry in the temperatures of the regenerator channels.

In Fig. 3, the temperature profile in the hottest channel of the regenerator appears to be more extreme than that in the coldest channel. This can be explained by the circulating flow causing approximately the same magnitude of energy transport for both channels while the oscillating mass flow had a smaller amplitude in the hottest channel due to the lower density of the gas. Therefore the axial mechanisms that balance the circulating flow must do so with a smaller oscillating mass flow in the hottest channel.

3.2. Energy transport due to circulation and changes in the energy stored in the matrix

The results from test to determine if the circulating flow could be the main energy transport mechanism increasing the asymmetry are shown in Fig. 5 shows that the rates of change in the energy stored in the matrices in the regenerator channels are smaller than the estimated rates of energy transport due to the circulating flow. The differences between the estimated and the actual rates are moderate in the beginning of the simulation ($f \cdot t < 5000$ in Fig. 5) and then become larger as the asymmetry increases. The differences are largest at the asymmetric periodic steady state solution ($f \cdot t > 30000$ in Fig. 5), where the time averages of the amounts of energy stored in the matrix channels are constant. Fig. 5 shows that the energy transport due to the circulating flow was large enough to be the mechanism that increases the asymmetry.

3.3. Contribution to the circulating flow from the pressure difference wave form

The results for how the instantaneous mass flow rate in the simple regenerator model depends on the temperature and pressure difference are shown in Fig. 6 shows that the mass flow rate for a given pressure difference increases with decreasing temperature and that this temperature dependence is largest for small pressure differences.

Fig. 6 shows that the shape of a pressure wave $\widetilde{\Delta p_{\mathrm{reg}}}$ that drives flow through two regenerator channels with different $\overline{T}$ can induce a circulating flow. Imagine, as the extreme case, that the pressure difference driving the flow through the regenerator channels looks like the square wave illustrated in Fig. 7. The shape of this pressure wave was calculated so that it gives zero cycle averaged mass flow at 200 K. In Fig. 7, the pressure difference is positive and of

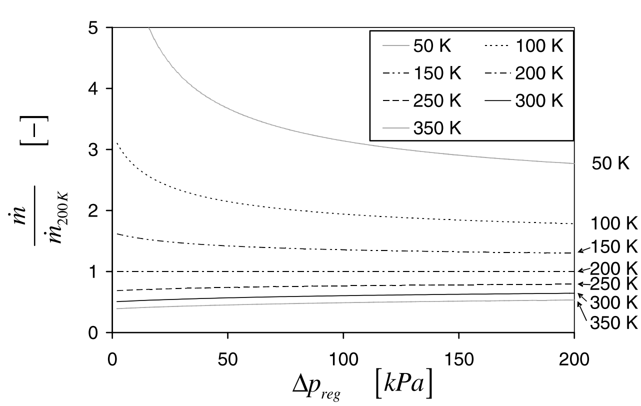

Fig. 6. Regenerator instantaneous mass flow rate relative to regenerator instantaneous mass flow rate at 200 K as function of the pressure difference between the ends of the regenerator.

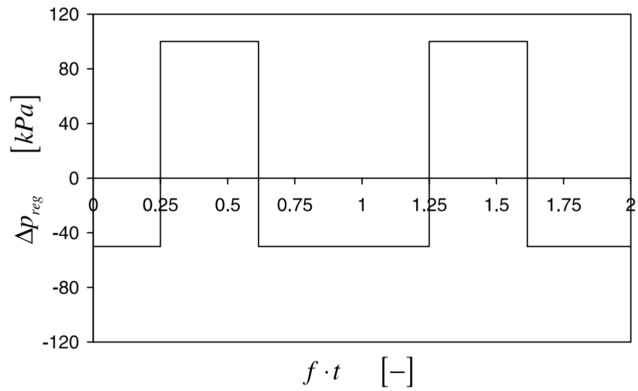

Fig. 7. Square wave pressure difference that gives zero cycle averaged flow in a regenerator channel at 200 K constant temperature and 2 MPa constant space averaged pressure.

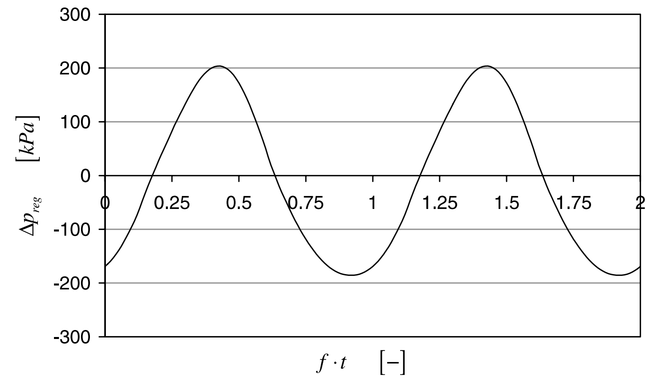

Fig. 8. Pressure difference over the regenerator versus time in the pulse tube cooler model. The pressure difference is positive when the pressure at the warm end is larger than the pressure at the cold end.

magnitude 100 kPa when it drives flow towards the cold end of the regenerator and negative and of magnitude 50 kPa when it drives flow towards the hot end. The pressure difference is negative during 63% of the cycle. If we apply this square wave to a regenerator channel where $\overline{T}$ is 100 K, then Fig. 6 shows that the flow towards the cold end would be 1.94 times larger (read at $\Delta p_{\mathrm{reg}} = 100$ kPa) and that flow towards the hot end would be 2.15 times larger (read at $\Delta p_{\mathrm{reg}} = 50$ kPa) than for $\overline{T} = 200$ K. At 100 K there would thus be a net cycle averaged mass flow rate, $f \cdot \oint \dot{m}$, towards the hot end. If we try with $\overline{T}$ larger than 200 K then the result is that there will be a larger cycle averaged mass flow towards the cold end.

Let us define the direction of the mass flow so that it is positive when it is towards the cold end. Then we can say that for the square wave pressure difference wave form from Fig. 7 and a regenerator with constant $\overline{T}$ and a constant $\overline{p}$ of 2 MPa, the derivative of $f \cdot \oint \dot{m}$ with respect to $\overline{T}$ is positive. If we the change sign on the pressure difference wave from Fig. 7, then the largest absolute pressure difference will drive flow towards the hot end. In this case the derivative of $f \cdot \oint \dot{m}$ with respect to $\overline{T}$ will be negative. For a sine wave shaped pressure difference then the derivative of $f \cdot \oint \dot{m}$ with respect to $\overline{T}$ is zero. The derivative of $f \cdot \oint \dot{m}$ with respect to $\overline{T}$ in the regenerator channel thus depends on the shape of the pressure difference wave $\widetilde{\Delta p_{\mathrm{reg}}}$.

Fig. 8 shows the pressure difference wave $\widetilde{\Delta p_{\mathrm{reg}}}$ over the regenerator from the complete PTC model plotted so that the pressure difference is positive when it drives flow towards the cold end of the regenerator. The pressure difference wave in Fig. 8 is nearly identical to the pressure difference wave predicted by the *Sage* model of the PTC. The curve in Fig. 8 shares some of the characteristics of the square wave in Fig. 7. The peak values of the positive pressure difference are 10% larger than the peak values of the negative pressure difference, and the pressure difference is negative during 54% of the cycle.

Fig. 9 shows the derivative of the net mass flow rate towards the cold end of a regenerator channel with respect to $\overline{T}$ for the pressure difference wave from Fig. 8 when neither $\overline{T}$ or $\overline{p}$ oscillate. The derivative is positive throughout the examined region and largest at low $\overline{T}$. This makes the symmetric situation unstable when there are more than one regenerator channel. Since the derivative of the cycle averaged mass flow rate with respect to $\overline{T}$ is largest at low $\overline{T}$ the situation will become increasingly unstable as the regenerator cools down.

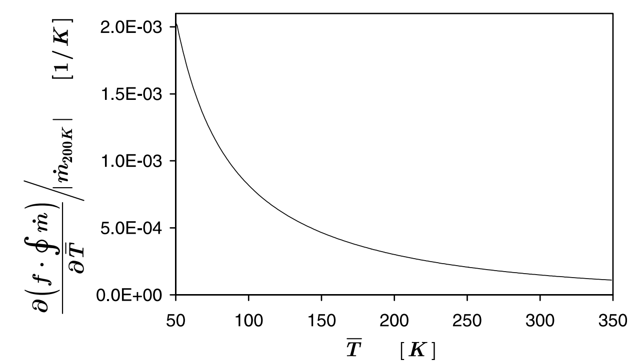

Fig. 9. Derivative of cycle averaged mass flow rate with respect to space averaged temperature $\overline{T}$ calculated at 2 MPa constant space averaged pressure calculated using the simple model of a regenerator. The derivative has been divided by the amplitude of the mass flow oscillation at 200 K in the simple regenerator model.

3.4. Effects of the oscillations in pressure and temperature, and the effect of the temperature dependent viscosity

The results from the test of the contributions to the cycle averaged mass flow rate from the pressure oscillation, temperature oscillation, and the effect of the temperature dependent viscosity are shown in Fig. 10.

At an average regenerator temperature of 200 K the 12% pressure oscillation increased $f \cdot \oint \dot{m}$ towards the cold end of a regenerator channel by approximately 4.5% of the amplitude of the mass flow oscillation.

Fig. 10 shows, that the pressure oscillation decreased the derivative with respect to $\overline{T}$ of $f \cdot \oint \dot{m}$ towards the cold end of a regenerator channel. The contribution to $f \cdot \oint \dot{m}$ from

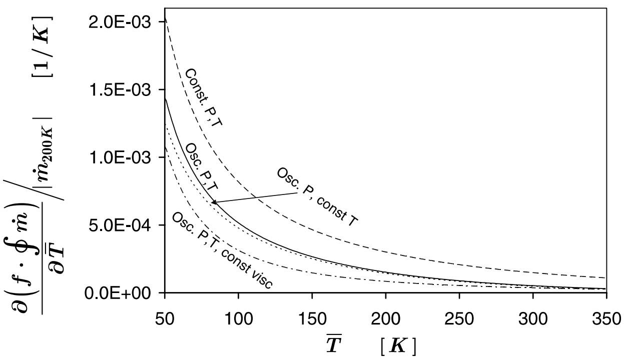

Fig. 10. Derivatives of cycle averaged mass flow rate with respect to space averaged temperature for different combinations of oscillating and constant temperatures and pressures, and constant or temperature dependent viscosity. The derivatives have been divided by the amplitude of the mass flow oscillation at 200 K in the simple regenerator model.

the oscillation in $\overline{p}$ hence had a stabilising effect when there was transverse asymmetry in the temperatures of the regenerator channels, and it did not contribute to the instability of the temperature profiles. Fig. 10 shows that the derivative with respect to $\overline{T}$ of $f \cdot \oint \dot{m}$ towards the cold end of a regenerator channel was reduced by 40–75%, but that it remained positive. The stabilising effect of the oscillation in $\overline{p}$ was therefore not large enough to remove the instability.

At an average regenerator temperature of 200 K the 1.4 K gas temperature oscillation decreased the cycle averaged mass flow through a regenerator channel towards the cold end of the regenerator by a mere 0.1% of the amplitude of the mass flow oscillation.

Fig. 10 shows that temperature oscillation slightly increased the derivative with respect to $\overline{T}$ of $f \cdot \oint \dot{m}$ towards the cold end of a regenerator channel. That is because the magnitudes of the derivatives of both μ and ρ with respect to temperature decrease with increasing temperature, and hence the effect due to the oscillation in $\overline{T}$ also decreases with increasing temperature. The temperature oscillation thus contributes to the instability, but the contribution is small.

Fig. 10 shows that removing the temperature dependence of the viscosity reduced the magnitude of the derivative with respect to $\overline{T}$ of $f \cdot \oint \dot{m}$ towards the cold end of a regenerator channel, but that the derivative remained positive. This shows that the temperature dependence of the viscosity contributed to the instability but that even with constant viscosity the flow resistance predicted by Eq. (1) would still lead to instability.

4. Generalisation of the results

In the complete PTC model there was a temperature gradient of 3600 K/m in the regenerator. In a short control volume in the regenerator, however, the variation in temperature inside the control volume was small compared to the average temperature in the control volume. For such a control volume the result of the above analysis, that used a space averaged temperature, should be valid, i.e. the control volume should exhibit the same tendencies as the single control volume of the simple regenerator model.

The effects driving the circulating flow were studied using Eq. (1) with coefficients for data by Gedeon and Wood [7] for flow through wire screen matrices to compute the flow friction in the regenerator. Thomas and Pitmann [13] provide eight sets of coefficients for Eq. (1) corresponding to experimental data and to correlations derived from experimental data from different authors for both wire screen and metal felt regenerator matrices. The curve in Fig. 10 for oscillating $\overline{T}$ and $\overline{p}$ was recalculated for each of the eight sets of coefficients provided by Thomas and Pitmann and there was only a slight spread between the curves for the different sets of coefficients. Hence the results of the above analysis should be valid for both felt and wire screen regenerators.

In this study the regenerator channels in the PTC model were identical and it was necessary to introduce the initial transverse asymmetry in the regenerator temperatures by hand (the notch in one of the channels). In real life it is impossible to create a perfectly symmetrical regenerator or PTC and hence some transverse asymmetry in the regenerator temperatures are to be expected. This transverse temperature asymmetry due to the geometrical asymmetry in a real PTC will hence always be available as a trigger for the temperature dependent effects discussed above.

5. Conclusions

A circulating flow that amplifies transverse asymmetry in the temperature profile in the regenerator of a pulse tube cooler has been reproduced using a complete simulation model of a pulse tube cooler where the regenerator is divided into two identical, parallel regenerator channels. The mechanisms governing the circulating flow have been identified and studied using a separate, simple model of a regenerator.

The primary mechanism driving the circulating flow was found to be due to the shape of the pressure difference wave and the dependence of the instantaneous mass flow rate on the instantaneous pressure difference between the ends of the regenerator and on the temperature in the regenerator. A small contribution to the circulating flow was also linked to the temperature oscillations in the regenerator. These mechanisms caused a regenerator channel to draw in mass from the cold end of the regenerator if the temperature in the channel decreased, and conversely to draw in more mass from the hot end of the regenerator if the temperature in the cannel increased. These mechanisms hence induced a circulating flow that amplified transverse asymmetry in the regenerator matrix temperature profiles. A mechanism due to the oscillations in pressure was found to have the opposite effect and hence worked against transverse asymmetry.

The asymmetry and the circulating flow increased the energy flux loss through the regenerator towards the cold heat exchanger by up to a factor of 5. The circulating flow was thus detrimental to the cooling power and the efficiency of a PTC.

It appears possible to reduce the magnitude of the asymmetry and hence of its detrimental effects on PTC performance by increasing transverse energy transport relative to axial energy transport in the regenerator. This can be done either by increasing the transverse heat conductivity of the regenerator matrix or by putting less heat load on the regenerator, so that the existing transverse heat conductivity is sufficient to keep the asymmetry at an acceptable level. It appears that dividing a single regenerator into two or more parallel regenerators with smaller cross sections is likely to maximise transverse asymmetry because it will inhibit the transverse energy transport that takes place in a single regenerator. It should thus lead to the largest circulating flow and the largest regenerator energy flux loss. Strong cooler losses have been experimentally observed by Kirkconnell in a specially designed small-size PTC with three parallel regenerator tubes [15].

Finally, it should be noted, that the shape of the pressure difference wave depends both on the design and the operating conditions of the PTC. It may be possible to sufficiently modify the shape of the pressure difference wave by changing the design or the operating conditions of the PTC in such a way that the tendency to induce circulating flow and instability is reduced.

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